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On Holographic Models of Memory

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Abstract

By means of the Volterra series an extension of the association properties of holography is proposed. Also, the significance of non-linearities embedded within the formal structure of a holographic-like associative memory is pointed out.

Starting from the holophon concept (Longuet-Higgins, 1968), Gabor (1968) suggested a temporal analogue model of the association properties of holography. Its formal structure consists of a sequence of convolutions and correlations which is characteristic for holography. In this connection it is interesting to notice that the most meaningful aspect of all the analogies recently brought up between brain functioning and holographic-like information processing is not directly connected with the physical mechanisms involved but rather with the underlying mathematical structure (Borsellino *et al.*, 1972).

To associate a temporal function $A(t)$ with $B(t)$ one performs and stores the convolution of A with B. To recall B it is enough to crosscorrelate the record with a fragment A' of A, provided that the function A is sufficiently complicated. More precisely, the Gabor model requires the record of

$$\phi_{BA}(\tau) = B * A = \int B(\xi)\, A(\tau - \xi)\, d\xi \tag{1}$$

and, in the recall phase, the crosscorrelation

$$F(t) = \int A'(x)\, \phi_{BA}(x+t)\, dx = A' \circledast \phi_{BA} = B * (A' \circledast A). \tag{2}$$

If $A' \circledast A \cong \delta$ (i.e. A is a "noiselike" function), $F(t) \cong B(t)$. Clearly this scheme is formally equivalent to Pavlovian conditioning. In the following I want to point out a possible generalization of this scheme. The first step [Eq. (1)] can be described as a linear filtering of the "signal". This description has been applied (Borsellino *et al.*, 1972) to show that Reichardt's correlation model for optomotor reactions is equivalent to a kind of short-term holophon-like memory.

Let us now consider the more general case of a non-linear interaction between A and B, as

$$\psi_{BA} = B \circ A\,. \tag{3}$$

It is possible for a wide class of cases to expand ψ_{BA} into a Volterra-Wiener series (Bedrosian *et al.*, 1971; Lee *et al.*, 1965) as

$$\begin{aligned} \psi_{BA}(t) &= \sum_{n=0}^{\infty} \frac{1}{n!} G_n[B_n, A(t)] \\ G_0 &= B_0 \\ G_1 &= \int B_1(\tau_1)\, A(t-\tau_1)\, d\tau_1 \\ G_2 &= \int\int B_2(\tau_1, \tau_2)\, A(t-\tau_1)\, A(t-\tau_2)\, d\tau_1\, d\tau_2 - K' \\ G_3 &= \ldots \end{aligned} \tag{4}$$

where B_1 represents the first (linear) kernel of the non-linear operation "B operating on A", and the other kernels are associated with higher order non-linearities; K' is a number dependent upon B_2 and A (see Lee). Let us now define the following operation

$$f \circledast^i g = \int g(t)\, f(t-\tau_1) \ldots f(t-\tau_i)\, dt\,, \tag{5}$$

where $\circledast^i$ is an extension of the usual crosscorrelation. If A (as A') is a noiselike function (ideally gaussian white noise with $A \otimes A = \delta$) the following result holds

$$F = A' \circledast^i \psi_{BA} = B_i(\tau_1 \ldots \tau_i) \tag{6}$$

except when two or more τ_i are equal

because of the orthogonal properties of these noiselike functions (see Lee). In this way if A and A' are complicated enough to be approximable by gaussian white noise, it becomes possible to retrieve selectively, by means of multiple correlations, all the information about the non-linear process which associates B to A. Clearly, what equations of the kind of Eq. (6) recall is in general not B but a coded form of B determined by the non-linear transformation Eq. (3). Interestingly enough a "one-dimensional" non-linearity [Eq. (4), equivalent to single input-output filtering] can provide directly, through Eq. (5), a set of multidimensional functions. Such a property may be interesting in studying the holographic-like associative behaviour of non-linear systems. The scheme embedded in

Eqs. (5)–(6) can also represent a simple memory which, for every k-dimensional set of values $(\tau_1 \ldots \tau_k)$, gives one value at the output $B_k(\tau_1, \ldots, \tau_k)$. The noiselike key signal $A' \equiv A$, which in this case carries no information, can be imagined to feed both the non-linear filter B and a set of delay lines controlled by the parameters $(\tau_1 \ldots \tau_i)$; the outputs are then cross-correlated according to equations like Eq. (6). In a similar scheme, one can envisage the possibility that a non-linearity, made "meaningful" through evolution-like processes, may play the role of "memory" in a nervous system.

The significant fact emerges from these considerations that strongly non-linear processes, embedded directly within the typical holographic structure, might improve considerably the effectiveness of such an associative memory. In addition, what has been outlined here suggests a general class of non-linear associative memories obtained by an extension of the holographic scheme through the Volterra-Wiener series. The problem may be formulated in quite general terms (Poggio, 1973).

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