Analysis of Reproducing Kernel Spaces in Learning Theory

Ding-Xuan Zhou

Dept. of Mathematics
City University of Hong Kong
Email: mazhou@math.ciyu.edu.hk

Learning Theory studies learning objects from random samples. The main question is: How many samples do we need to ensure an error bound with certain confidence? To answer this question, estimates for the approximation error and the sample error involving covering numbers or entropy numbers play an essential role.

For kernel machine learning such as the Support Vector Machine, a Reproducing Kernel Hilbert Space associated with a Mercer kernel $K$ is often used and a ball $B_R$ of such a space (as a subset of $C(X)$) is taken as the hypothesis space.

In this talk we shall first consider the approximation error for the regression problem. The rate of convergence of the target function from $B_R$ to the regression function (as the radius $R$ becomes large) will be estimated by means of the regularity of the regression function and the kernel.

Then we shall estimate the covering number of the set $B_R$. Our estimates are based on the regularity of the kernel function. For convolution type kernels $K(x, t) = k(x - t)$ with $k$ being analytic, we provide estimates depending on the decay of the Fourier transform of $k$. For general kernels, we show that there producing kernel Hilbert space can be embedded into $C^{s/2}$ if $K$ is $C^s$. Hence some estimates for the covering number follow. Our analysis provides estimates for the approximation error and the sample error associated with many important kernels in Learning Theory.