

NSM 00445

A simple algorithm for solving the cable equation in dendritic trees of arbitrary geometry

Christof Koch and Tomaso Poggio

Center for Biological Information Processing and Artificial Intelligence Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139 (U.S.A.)

(Received April 16th, 1984)

(Revised August 16th, 1984)

(Accepted October 1st, 1984)

Key words: neuronal modeling – cable theory – algorithm – Laplace domain – Green function – dendritic tree – synaptic input – quasi-active membrane – retinal ganglion cells – dendritic spines

We present an efficient algorithm for solving the one-dimensional cable equation in the Laplace (frequency) domain for an arbitrary linear membrane. This method, a reformulation and extension of the geometrical calculus developed by Butz and Cowan (1974), solves for the transfer impedance between any two points in a branched cable structure of arbitrary geometry (but without loops) by the repetitive application of four simple equations. Such an algorithm is used to analyze the electrical behaviour of nerve cells with highly branched dendritic trees. The algorithm can be implemented using a language such as C, PASCAL or LISP and runs on small machines.

Introduction

The dendritic trees of different types of nerve cells can have very different morphologies. A case in point are the histological classes of ganglion cells in the cat retina (α , β , γ and δ ; see Boycott and Wässle, 1974). To understand how the dendritic morphology and the synaptic architecture influence the function of a neuron, it is important to calculate the effect of an active synapse on the membrane potential at the dendrite and at the soma. Such a study was initiated by Rall and co-workers (for a review see Rall, 1977) who investigated the solution of the cylindrical cable equation for different cable configurations. He discovered (1964) that under certain conditions (the diameters of the branches must satisfy the so-called $d^{3/2}$ law), a whole dendritic tree can be reduced to a single cylinder, called the equivalent cylinder, greatly simplifying the problem. The $d^{3/2}$ law, however, does

Correspondence: C. Koch, Center for Biological Information Processing and Artificial Intelligence Laboratory, Massachusetts Institute of Technology, 545 Technology Square, Cambridge, MA 02139, U.S.A.

not hold for a variety of neurons such as hippocampal pyramidal cells (Turner and Schwartzkroin, 1980), the α -motoneurons (Barrett and Crill, 1974) and retinal ganglion cells (Koch et al., 1982). Another way of analyzing extended neuronal structures is to segment the structure into small, lumped compartments, assumed to be electrically homogeneous, and to calculate the potential within each compartment (Rall, 1964; Perkel et al., 1981). This method suffers from the drawback that the required numerical calculations are computationally expensive. In addition, the solution represents only a discrete approximation to the continuous equation. Therefore, an efficient method for computing voltage transients in arbitrary structures is desirable. Butz and Cowan (1974) were able to derive an elegant geometrical calculus for the investigation of branching dendritic structures of arbitrary geometry by representing a cable with its passive membrane in the Laplace domain and solving for the transfer function $K_{ij}(\omega)$. Knowledge of the transfer-function allows one to compute the voltage response to arbitrary current inputs. We have implemented their method on a computer and applied it to an analysis of cat retinal ganglion cells to elucidate the relation between the dendritic morphology and the known electrical function of these cells (Koch et al., 1982, 1983) and to an analysis of the functional significance of dendritic spines (Koch and Poggio, 1983). We generalized Butz and Cowan's method to include cables with an arbitrary linear membrane impedance (Koch, 1984). This is useful, since the electrical properties of a variety of cells like the lamprey giant interneurons (Moore and Christensen, 1984), the rod photoreceptors in the retina (Detwiler et al., 1980; Torre and Owen, 1983), the hair cells of the cochlea (Crawford and Fettiplace, 1981) and the transverse tubular membrane of skeletal muscle (Moore and Tsai, 1983) can be modeled by a membrane consisting of inductance-like components arising from small signal linearization of time and voltage dependent conductances.

We report here an equivalent but simpler algorithm which permits the automatic computation of the transfer function in cable structures with arbitrary geometry and a generalized linear membrane by the recurrent application of four elementary equations. This algorithm is conceptually less complex and, if implemented recursively, more efficient in terms of computation time than our implementation of the Butz and Cowan scheme.

The general representation of a cable as a transmission line

The most important assumption underlying 1-dimensional cable theory is linearity of the dendritic membrane, i.e. the membrane impedance z_m depends on time but not on the applied voltage. To solve the problem of deriving the Green function for a cable with a linear—but not necessarily passive—membrane, we consider the cable equation in the frequency domain. This approach has the advantage of offering a general and simple solution to the cable equation in arbitrary cables.

We represent a linear, 1-dimensional cable as an infinite ladder network with arbitrary transverse and serial impedances $z_m(\omega)$ and $z_a(\omega)$ as in Fig. 1*. If the

* The radial frequency ω is defined as $2\pi f$ where f is the frequency in Hz.

membrane is assumed to be passive, $z_m(\omega)$ is modeled by a capacitance c_m in parallel with a resistance r_m :

$$z_m(\omega) = \frac{r_m}{1 + i\omega\tau_m} \quad (1a)$$

with the membrane time-constant $\tau_m = c_m r_m$. Analyzing the small-signal behavior of the squid axon, yields a membrane which is described by two inductances, two capacitances and 4 resistances (Sabah and Leibovic, 1969; Mauro et al., 1970). Thus, the associated membrane impedance is given by

$$z_m(\omega) = \frac{\alpha_3\omega^3 + \alpha_2\omega^2 + \alpha_1\omega + \alpha_0}{\beta_4\omega^4 + \beta_3\omega^3 + \beta_2\omega^2 + \beta_1\omega + \beta_0} \quad (1b)$$

The impedance of the intracellular cytoplasm is given by a ohmic resistance, although our algorithm does not require it:

$$z_a(\omega) = r_a \quad (2)$$

Under the basic assumptions of 1-dimensional cable theory (for a review consult Rall, 1977 and Jack et al., 1975) the equations can be written directly in the Laplace domain as

$$\frac{\partial V(x, \omega)}{\partial x} = -z_a(\omega) i_a(x, \omega) \quad (3)$$

$$\frac{\partial i_a(x, \omega)}{\partial x} = -i_m(x, \omega) \quad (4)$$

and

$$V(x, \omega) = z_m(\omega) i_m(x, \omega) \quad (5)$$

where $V(x, \omega)$, $i_a(x, \omega)$ and $i_m(x, \omega)$ are the Laplace transforms (with respect to time) of the membrane potential $V(x, t)$, the intracellular axial current $i_a(x, t)$ and the membrane current $i_m(x, t)$. Combining these equations leads to a general form of the cable equation for an infinite cable:

$$\frac{\partial^2 V(x, \omega)}{\partial x^2} = \gamma(\omega)^2 V(x, \omega) \quad (6)$$

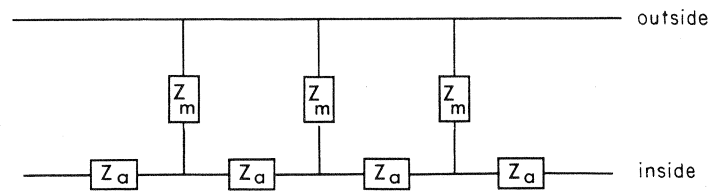


Fig. 1. The general representation of a one-dimensional cable with arbitrary membrane impedance $z_m = z_m(\omega)$ and serial impedance $z_a = z_a(\omega)$. The extracellular impedance $z_o(\omega)$ is neglected.

with the propagation constant $\gamma(\omega)$ given by

$$\gamma(\omega)^2 = \frac{z_a(\omega)}{z_m(\omega)} \quad (7)$$

For the steady-state case, $\omega = 0$, $\gamma^2 = r_a/r_m = 1/\lambda^2$, where λ is the space constant. Eqn. 6 shows the advantage of using the Laplace transform $V(x, \omega)$ instead of the original function. If the membrane contains a resistance, an inductance in series with another resistance and a capacitance, the cable equation in the time domain is a partial-differential equation of order three, while the corresponding Eqn. 6 is an ordinary second order differential equation. The nature of the neuronal membrane is fully contained in $\gamma(\omega)$ and does not affect the form of Eqn. 6.

If we follow Rall (1959) and Butz and Cowan (1974) in the choice of hyperbolic functions, the solution of Eqn. 6 has the general form

$$V(x, \omega) = A \cosh\{\gamma(\omega)x\} + B \sinh\{\gamma(\omega)x\} \quad (8)$$

where A and B depend on the frequency and boundary conditions. The next section shows how the voltage distribution in a cable structure of arbitrary geometry can be derived.

The four elementary rules

In the following, we list 4 elementary rules which uniquely determine the potential in an arbitrary cable structure (without loops) generated by a current input

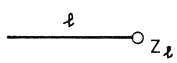

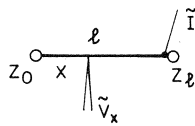
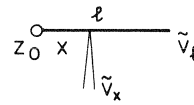
Rule I		$Z = Z_c \frac{Z_l \cosh(\gamma \cdot l) + Z_c \sinh(\gamma \cdot l)}{Z_l \sinh(\gamma \cdot l) + Z_c \cosh(\gamma \cdot l)}$
Rule II		$Z^{-1} = Z_1^{-1} + Z_2^{-1}$
Rule III		$\tilde{V}_x = Z_l \frac{Z_c \sinh(\gamma \cdot x) + Z_0 \cosh(\gamma \cdot x)}{(Z_0 + Z_l) \cosh(\gamma \cdot l) + (Z_c + Z_0 Z_l) \sinh(\gamma \cdot l)} \tilde{I}$
Rule IV		$\tilde{V}_x = \frac{Z_0 \cosh(\gamma \cdot x) + Z_c \sinh(\gamma \cdot x)}{Z_0 \cosh(\gamma \cdot l) + Z_c \sinh(\gamma \cdot l)} \tilde{V}_l$

Fig. 2. The 4 rules needed to solve the cable equation in one-dimension cables. The first rule gives the impedance of a single cylinder and the second the combined impedance of two branching cylinders. The third rule expresses the voltage at any point along a cylinder as a function of the injected current at one end. The fourth rule relates the voltage at any point in a cylinder to the voltage at one end. All variables are functions of the radial frequency ω .

(Fig. 2). The equations themselves are not new and can be found, in different forms, in different publications (Rall, 1959; Barrett and Crill, 1974; Butz and Cowan, 1974; Glasser, 1977). Our main contribution consists of the extension of these equations to cover generalized linear membranes and of their implementation in a fast and efficient algorithm that applies to arbitrary branched trees.

Z_c denotes the characteristic impedance of the specific branch considered:

$$Z_c(\omega) = z_a(\omega) / \gamma(\omega) \quad (9)$$

and $Z_0(\omega)$ and $Z_l(\omega)$ the terminal impedance at the left ($x = 0$) and the right ($x = l$) boundary of a given branch. Under the constraint that the potential tends to zero (killed end; see Rall, 1959) it follows that the terminal impedance Z_0 (or Z_l) is zero. The more realistic condition in which no current is allowed to cross the terminal boundary $\partial V / \partial x = 0$ (sealed end) implies $Z_0 \rightarrow \infty$.

Rule I describes the impedance $Z(\omega)$ of a branch of length l and terminal impedance Z_l :

$$Z(\omega) = Z_c(\omega) \frac{Z_l(\omega) \cosh\{\gamma(\omega)l\} + Z_c(\omega) \sinh\{\gamma(\omega)l\}}{Z_l(\omega) \sinh\{\gamma(\omega)l\} + Z_c(\omega) \cosh\{\gamma(\omega)l\}} \quad (10)$$

If the branch is a terminal branch, satisfying the sealed-end boundary condition, Eqn. 10 reduces to:

$$Z(\omega) = Z_c(\omega) \frac{\cosh\{\gamma(\omega)l\}}{\sinh\{\gamma(\omega)l\}}$$

These equations are given in Butz and Cowan (1974; Eqn. 52) and can easily be derived using four-pole transmission-line theory as set forth in Weber (1965).

Rule II gives the total impedance of a branch point. If two branches at a fork have input impedance $Z_1(\omega)$ and $Z_2(\omega)$, respectively, their combined admittance is the sum of the individual admittances, i.e.

$$Z(\omega)^{-1} = Z_1(\omega)^{-1} + Z_2(\omega)^{-1} \quad (11)$$

Notice that this rule can be generalized immediately to an arbitrary number of cylinders branching off.

Rule III gives the voltage at position x (with respect to the local coordinate system of the branch) if a current $I(\omega)$ is injected at one end of the cable ($x = l$):

$$\begin{aligned} V(x, \omega) = & Z_l(\omega) [Z_c(\omega) \sinh\{\gamma(\omega)x\} + Z_o(\omega) \cosh\{\gamma(\omega)x\}] I(\omega) \\ & / [(Z_0(\omega) + Z_l(\omega)) \cosh\{\gamma(\omega)l\} + (Z_c(\omega) + Z_0(\omega)Z_l(\omega)/Z_c(\omega)) \\ & \times \sinh\{\gamma(\omega)l\}] \end{aligned} \quad (12)$$

Eqn. 12 is given in Butz and Cowan (1974; Eqn. 3).

Rule IV gives the voltage at position x along the branch if the potential is known to be $V_l(\omega)$ at one end of the cylinder ($x = l$):

$$V(x, \omega) = \frac{Z_c(\omega) \sinh\{\gamma(\omega)x\} + Z_0(\omega) \cosh\{\gamma(\omega)x\}}{Z_c(\omega) \sinh\{\gamma(\omega)l\} + Z_0(\omega) \cosh\{\gamma(\omega)l\}} V_l(\omega) \quad (13)$$

This equation can be obtained by inverting Eqn. 12 to give the current $I(\omega)$ if the voltage at one end ($x = l$) is held at $V(\omega)$. Inserting $I(\omega)$ into Eqn. 11 yields directly Eqn. 13.

If the injected current in Eqn. 12 is a δ pulse, i.e. $I(\omega) = 1$, the function $V(x, \omega)$ can be identified with the transfer-function or transfer-impedance $K_{ij}(\omega)$, where i and j are the input-output locations*. If these two locations coincide, i.e. $x = l$ in Eqn. 12, the familiar input impedance $K_{ii}(\omega)$ is obtained. Knowledge of the transfer impedance between i and j allows one to compute the voltage response at j , $V_j(\omega)$, to an arbitrary current input $I_i(\omega)$ at i ,

$$V_j(\omega) = K_{ij}(\omega) \cdot I_i(\omega) \quad (14a)$$

or, in the time-domain

$$V_j(t) = K_{ij}(t) * I_i(t) \quad (14b)$$

where $*$ represents convolution and $K_{ij}(t)$ is the Green-function of the system, i.e. the Laplace-transform of $K_{ij}(\omega)$.

In deriving the transfer-impedance $K_{ij}(\omega)$ in a branched tree structure, our program reduces the dendritic tree to a configuration amenable to rule III by applying repetitively rules I and II until the whole structure is reduced to a single branch with two terminal impedances, at one end of which the current is injected. After a single invocation of rule III, the transfer-impedance to any point in the cable structure can be calculated via recurrent application of rule IV by unfolding the tree (see Fig. 3).

Treating synaptic input as conductance change

Synaptic inputs, however, consist of transient conductance changes $g(t)$ to specific ions with equilibrium potential E , relative to the resting potential of the cell. The resulting current $I_i(t)$ at location i is, in general, not proportional to $g(t)$, since

$$I_i(t) = g(t) \cdot (E - V_i(t)) \quad (15)$$

Convolving both sides with the input impedance $k_{ii}(t)$ yields

$$V_i(t) = K_{ii}(t) * [g(t) \cdot \{E - V_i(t)\}] \quad (16)$$

a Volterra integral equation (Rall, 1964; Poggio and Torre, 1978, 1981). The nonlinear nature of the synaptic input is crucial for a number of specific computa-

* Thus, the absolute value of $K_{ij}(\omega)$ has the dimension of a resistance.

tions, such as direction-selectivity (Torre and Poggio, 1978) and activity-dependent changes in spine morphology (Koch and Poggio, 1983). Thus, once $K_{ii}(t)$ has been obtained, Eqn. 16 can easily be solved by simple numerical integration (Koch et al.,

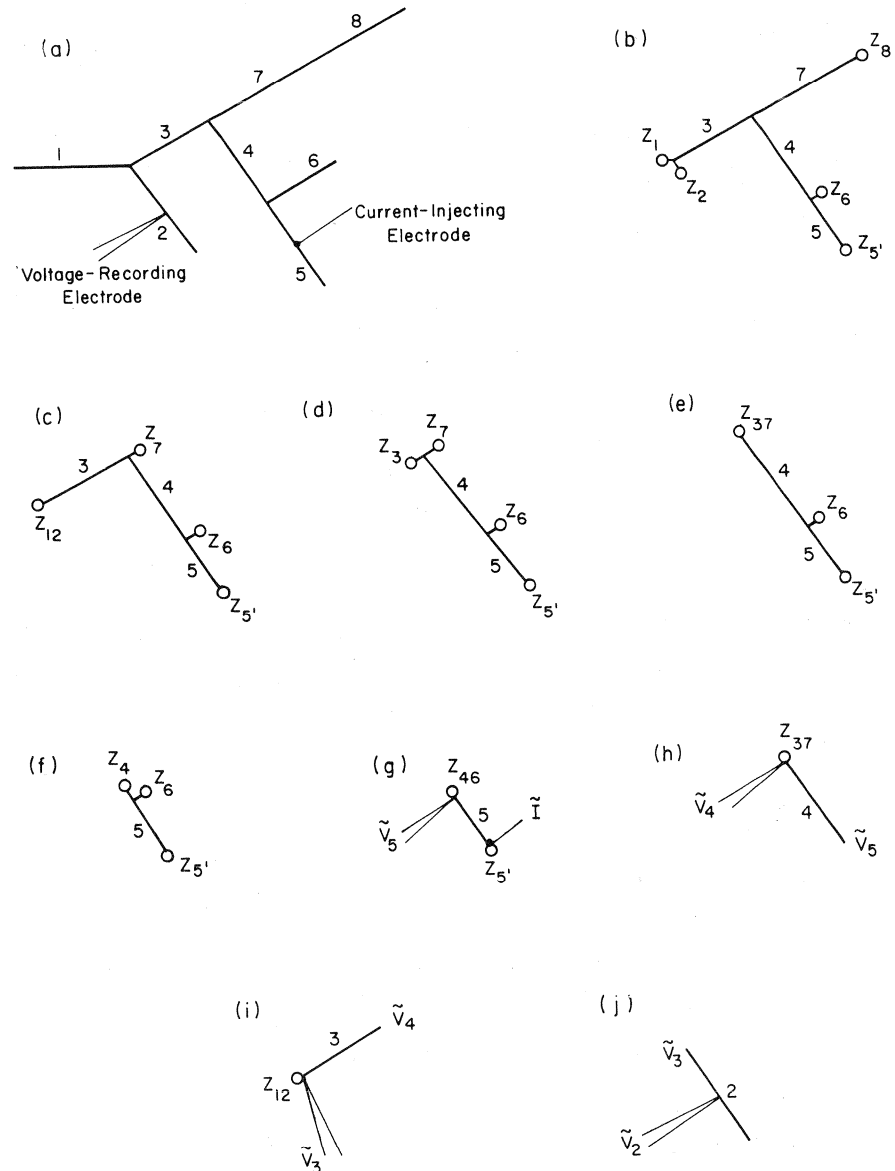


Fig. 3. The application of the algorithm to a simple neuron with the current source in branch 5 and the voltage-recording electrode in branch 2. In a to f the tree is reduced by recurrent application of rules I and II to a single cylinder. In f to j the potential due to the injected current $I(\omega)$ is "propagated" back towards branch 2 (see text).

1983) to give the voltage $V_i(t)$. Note that the optimal time-step, Δt , used for the integration, depends on the input site and the time-course of $g(t)$.

The voltage evoked by the synaptic input, for instance at the soma, is subsequently determined using Eqn. 14b. When computing the voltage response to two or more simultaneous synaptic inputs, the nonlinear interaction between synapses has to be taken into account (Koch et al., 1982, 1983).

An example

We will consider a simple neuron, represented by 8 cylinders (Fig. 3a), with passive membrane R_m . For the sake of our argumentation, we will identify branch 3 with the soma, i.e. a very short but thick diameter, branches 2, 4 and 7 with primary dendrites and branch 1 with the axon. Since we are considering only the passive behavior of the cell, the axon is modeled as a very long cable with lowered membrane resistance R_{m1} . The membrane capacity C_m and the intracellular resistance R_i are assumed to be uniform throughout the cell. As in all of our previous studies, terminal impedances (in our case, Z_1 , Z_2 , Z_5 , Z_6 and Z_8) obey the sealed end boundary condition, i.e. $Z_l \rightarrow \infty$.

$Z_c(\omega)$ for branch 1 is given by $r_{a1}/\gamma_1(\omega)$, where $\gamma_1(\omega)^2 = r_{a1}(1 + i\omega r_{m1}c_{m1})/r_{m1}$. The values r_{m1} , c_{m1} and r_{a1} are derived from the specific membrane parameters using $r_{m1} = R_m/(\pi d_1)$, $c_{m1} = \pi d_1 C_m$ and $r_{a1} = 4R_i/d_1^2$, where d_1 is the diameter of branch 1 (Jack et al., 1975)*. The values for all other branches are computed in a similar fashion. The lengths l_i are given in cm. We will determine the input impedance at x_5 in branch 5 (Fig. 3a) and the transfer impedance between that location and x_2 in branch 2.

In a first series of steps our computer program reduces the dendritic tree to a configuration needed to apply rule III, that is it 'folds' the tree into a single branch. Applying rule 1 for the sealed end, we obtain

$$Z_1(\omega) = Z_{c1}(\omega) \frac{\cosh\{\gamma_1(\omega)l_1\}}{\sinh\{\gamma_1(\omega)l_1\}}$$

$$Z_2(\omega) = Z_{c2}(\omega) \frac{\cosh\{\gamma_2(\omega)l_2\}}{\sinh\{\gamma_2(\omega)l_2\}}$$

$$Z'_5(\omega) = Z_{c5}(\omega) \frac{\cosh\{\gamma_5(\omega)(l_5 - x_5)\}}{\sinh\{\gamma_5(\omega)(l_5 - x_5)\}}$$

and likewise for $Z_6(\omega)$ and $Z_8(\omega)$. The tree of Fig. 3a is now reduced to Fig. 3b. Making use of rule II: $Z_{12}(\omega)^{-1} = Z_1(\omega)^{-1} + Z_2(\omega)^{-1}$ and once again rule I:

$$Z_7(\omega) = Z_{c7}(\omega) \frac{Z_8(\omega) \cosh\{\gamma_7(\omega)l_7\} + Z_{c7} \sinh\{\gamma_7(\omega)l_7\}}{Z_8(\omega) \sinh\{\gamma_7(\omega)l_7\} + Z_{c7} \cosh\{\gamma_7(\omega)l_7\}}$$

* Note that R_m is given in Ωcm^2 , C_m in Fcm^{-2} and R_i in Ωcm .

gives Fig. 3c. Repetitive application of rules I and II (see Fig. 3d–g) leads to the configuration depicted in Fig. 3g. For the input impedance $K_{55}(\omega)$ we make use of rule III and remember that $I(\omega) = 1$

$$K_{55}(\omega) = Z_{46}(\omega) [Z_{c5}(\omega) \sinh\{\gamma_5(\omega)x_5\} + Z'_5(\omega) \cosh\{\gamma_5(\omega)x_5\}] \\ / [(Z_{46}(\omega) + Z'_5(\omega)) \cosh\{\gamma_5(\omega)l_5\} \\ + (Z_{c5}(\omega) + Z_{46}(\omega)Z'_5(\omega)/Z_{c5}(\omega)) \sinh\{\gamma_5(\omega)l_5\}]$$

In order to compute the transfer impedance $K_{52}(\omega)$, the program ‘unfolds’ the tree by use of rule IV (Fig. 3h, i and j):

$$K_5(\omega) = [Z_{46}(\omega)Z'_5(\omega)] / [(Z_{46}(\omega) + Z'_5(\omega)) \cosh\{\gamma_5(\omega)l_5\} + (Z_{c5}(\omega) \\ + Z_{46}(\omega)Z'_5(\omega)/Z_{c5}(\omega)) \sinh\{\gamma_5(\omega)l_5\}]$$

$$K_4(\omega) = \frac{Z_{37}(\omega)K_5(\omega)}{Z_{37}(\omega) \cosh\{\gamma_4(\omega)l_4\} + Z_{c4}(\omega) \sinh\{\gamma_4(\omega)l_4\}}$$

$$K_3(\omega) = \frac{Z_{12}(\omega)K_4(\omega)}{Z_{12}(\omega) \cosh\{\gamma_3(\omega)l_3\} + Z_{c3}(\omega) \sinh\{\gamma_3(\omega)l_3\}}$$

and, finally, taking account of the sealed end terminal in branch 2 (Fig. 3j):

$$K_{52}(\omega) = K_3(\omega) \frac{\cosh\{\gamma_2(\omega)x_2\}}{\cosh\{\gamma_2(\omega)l_2\}}$$

Once the complex transfer-function $K_{ij}(\omega)$ has been computed, the Green function $K_{ij}(t)$ is obtained by inverse Fourier transformation using for instance the widely available ‘Fast Fourier Transform’ (FFT) algorithm derived by Cooley and Tukey (1965). For details of the inversion, along with a discussion of the resulting errors of approximation and aliasing, see Norman (1972). The voltage at 2 and 5 for the current input $I_5(t)$ is given by $V_5(t) = K_{55}(t) * I_5(t)$ and $V_2(t) = K_{52}(t) * I_5(t)$.

However, if one is only interested in the steady-state properties of neurons, when most nonlinear conductances are presumably inactivated, one can forego computing the transfer-function for a large frequency spectrum. For a stationary current input I_i , the real steady-state transfer-impedance $K_{ij}(\omega = 0) = K_{ij}$ gives directly the induced voltage change $V_j = K_{ij} \cdot I_i$. For a stationary synaptic conductance input g_i , or for a conductance change with a time-course several times longer than the membrane time-constant τ_m (Koch et al., 1983), V_i is given by $V_i = g_i EK_{ii} / (1 + g_i K_{ii})$. Computing the response of the neuron to long-lasting synaptic inputs can therefore be done with considerably reduced computational overhead.

Use of a recursive, high-level computer language, such as PASCAL, C or LISP, leads to a simple implementation of the algorithm for computing $K_{ij}(\omega)$ in arbitrary

* Two properties of K_{ij} can be exploited to synthesize transfer functions: $K_{ij} = K_{ji}$ and $K_{ij} = K_{ii} \cdot K_{ij} / K_{ii}$ for any point l lying between i and j (Koch et al., 1982).

dendritic trees*. The structure of the neuron is described by considering the branches as nodes and associating with each such node three pointers or edges: one always points to the parent branch (i.e. the branch where the current branch originated), while the other two point to any existing daughter branches (if no daughter branches exist, such as in the case of a terminal branch, the corresponding pointers are set to nil). The program requires as input the branching pattern of the neuron and the parameters describing the dendritic membrane. In the most general case, the user can specify a different type of membrane for every branch or parts thereof. It is therefore straightforward to model cells with an inhomogeneous distribution of the membrane resistance R_m throughout the dendritic tree (as postulated for α -motoneurons; Fleshman et al., 1983). The program then automatically computes the transfer function K_{ij} for a specified frequency range between any two points.

Discussion and comparison with other methods

The major advantage of our algorithm is its generality. Our algorithm computes for any neuronal geometry the Laplace transform of the voltage response at any point j in response to a current input at location i . This compares well with most other methods giving either an approximation of the voltage response in spatially inhomogeneous structures such as all compartment methods (for a review of these see Perkel et al., 1981), introduce extra assumptions, like Rall's equivalent cylinder method (Rall, 1964), or consider only very simple cable configurations as Jack and Redman (1971) or Norman (1972) have done. The method first proposed by Rall (1959) and subsequently applied by Barrett and Crill (1974) to determine the input impedance of a reconstructed α -motoneuron can be regarded as a special case of our approach. More similar to our method is the system used by Glasser and colleagues (Glasser et al., 1977, Glasser, 1977) for describing the electrical properties of the lobster stomatogastric ganglion. Note that recently, a slightly different derivation of a similar algorithm has appeared (Turner 1984a, b).

As we have already pointed out in the introduction, Butz and Cowan (1974) developed a geometrical calculus for computing the Laplace transform of the voltage in dendritic systems of arbitrary geometry. We have extended their results to cover arbitrary linear membranes and cylinders with discontinuous change in diameter and membrane properties and implemented them into a PASCAL program. This algorithm, while giving the same results, is conceptually more elaborate than the algorithm which we have presented. This increased complexity is mirrored in a longer computation time**.

* A listing of a documented LISP program will be supplied upon request. A PASCAL version of the program has been implemented on the IBM Personal Computer (E. Famiglietti, personal comm.).

** About a factor of 5; for a dendritic tree consisting of 135 branches, the present implementation of our algorithm on a single user, medium size computer (Lisp Machine) requires about 4 s to compute $K_{ij}(\omega)$ for a given ω .

These frequency domain methods have specific advantages and disadvantages. Their main advantage is that once the transfer function K_{ij} has been computed, the voltage response at j for arbitrary current or conductance inputs at i can be easily obtained, unlike compartments methods. Moreover, $K_{ij}(\omega)$ is, for a specified neuronal geometry, the exact solution of the cable equation. It is only during the inverse Laplace transform into the time-domain, that errors are introduced. This allows the simulation of anatomically and physiologically discrete inputs to the cell. Another point in their favor is the possibility of directly matching the response of a neuron to a small amplitude white noise signal with the computed $K_{ij}(\omega)$ (*white noise analysis*; Marmarelis and Marmarelis, 1978). Voltage clamping the intracellular potential in neurons with extended dendrites and subsequent perturbation of the system allows the study of voltage dependent conductances by piecewise linearization, providing the kinetic parameters of the underlying channels. Such an approach has been used, for instance, to study cable properties of cultured neurons and of lamprey central neurons (Moore and Christensen, 1984). A further application of our approach is an analysis of the effect of linear membranes having inductance-like properties on the information processing performed in dendritic trees (Koch, 1984).

The most significant drawback of our method is its inability to treat nonlinear membranes. Active behaviour involving spike initiation and propagation cannot be modeled. This constraint limits the application of this class of algorithms to non-spiking neurons, dendritic trees with passive properties or to a small-signal analysis of neuronal structures with active structures, such as the squid axon (Mauro et al., 1970). Modeling nonlinear electric behavior necessitates the use of more general programs, such as the electric circuit simulation program SPICE, recently adopted to neuronal modeling (Segev et al., 1984).

Acknowledgments

C.K. is supported by the Fritz Thyssen foundation. Support for T.P. is provided in part by the Sloan Foundation and the Whitaker College and by the Artificial Intelligence Laboratory, a research program supported in part by the Advanced Research Project Agency of the Department of Defense under Office of Naval Research contract N00014-80-C-0505.

References

- Barrett, J.N. and Crill, W.E. (1974) Specific membrane properties of cat motoneurons, *J. Physiol. (Lond.)*, 239: 301–324.
- Boycott, B.B. and Wässle, H. (1974) The morphological types of ganglion cells of the domestic cat's retina, *J. Physiol. (Lond.)*, 240: 397–419.
- Butz, E.G. and Cowan, J.D. (1974) Transient potentials in dendritic systems of arbitrary geometry, *Biophys. J.*, 14: 661–689.

- Cooley, J.W. and Tukey, J.W. (1965) An algorithm for the machine calculation of complex Fourier series, *Math. Comp.*, 19: 297-301.
- Crawford, A.C. and Fettiplace, R. (1981) An electrical tuning mechanism in turtle cochlear hair cells, *J. Physiol. (Lond.)*, 312: 377-412.
- Detwiler, P.B., Hodgkin, A.L. and McNaughton, P.A. (1980) Temporal and spatial characteristics of the voltage response of rods in the retina of the snapping turtle, *J. Physiol. (Lond.)*, 300: 213-250.
- Fleshman, J.W., Segev, I., Cullheim, S. and Burke, R.E. (1983) Matching electrophysiological with morphological measurements in cat α -motoneurons, *Soc. Neurosci. Abstr.*, 9: 341.
- Glasser, S. (1977) Computer Reconstruction and Passive Modeling of Identified Nerve Cells in the Lobster Stomatogastric Ganglion, Ph.D. Thesis, University of California San Diego.
- Glasser, S., Miller, J., Xuong, N.G. and Selverston, A. (1977) Computer reconstruction of invertebrate nerve cells. In R.D. Lindsay (Ed.), *Computer Analysis of Neuronal Structures*, Plenum Press, New York, pp. 21-58.
- Jack, J.J.B., Noble, D. and Tsien, R.W. (1975) *Electric Current Flow in Excitable Cells*, Clarendon Press, Oxford.
- Jack, J.J.B. and Redman, S.J. (1971) An electrical description of the motoneurone and its application to the analysis of synaptic potentials, *J. Physiol. (Lond.)*, 215: 321-352.
- Koch, C. (1984) Cable theory in neurons with active, linearized membranes, *Biol. Cybern.*, 50: 15-33.
- Koch, C. and Poggio, T. (1983) A theoretical analysis of electrical properties of spines, *Proc. roy. Soc. B*, 218: 455-477.
- Koch, C., Poggio, T. and Torre, V. (1982) Retinal ganglion cells: a functional interpretation of dendritic morphology, *Phil. Trans. roy. Soc. B*, 298: 227-264.
- Koch, C., Poggio, T. and Torre, V. (1983) Non-linear interaction in a dendritic tree: localization, timing and role in information processing, *Proc. nat. Acad. Sci. U.S.A.*, 80: 2799-2802.
- Marmarelis, P.Z. and Marmarelis, V.Z. (1978) *Analysis of Physiological Systems: The White-Noise Approach*, Plenum Press, New York.
- Mauro, A., Conti, F., Dodge, F. and Schor, R. (1970) Subthreshold behavior and phenomenological impedance of the squid giant axon, *J. gen. Physiol.*, 55: 497-523.
- Moore, L.E. and Christensen, B.N. (1984) White noise analysis of cable properties of neuroblastoma cells and lamprey central neurons, Submitted.
- Moore, L.E. and Tsai, T.D. (1983) Ion conductances of the surface and transverse tubular membranes of skeletal muscle, *J. Memb. Biol.*, 73, 217-226.
- Norman, R.S. (1972) Cable theory for finite length dendritic cylinders with initial and boundary conditions, *Biophys. J.*, 12: 25-45.
- Perkel, D.H., Mulloney, B. and Budelli, R.W. (1981) Quantitative methods for predicting neuronal behavior, *Neuroscience*, 6: 823-837.
- Poggio, T. and Torre, V. (1978) A new approach to synaptic interaction. In R. Heim and G. Palm (Eds.), *Theoretical approaches to complex systems, Lecture notes in biomathematics*, 21, Springer, Heidelberg, pp. 89-115.
- Poggio, T. and Torre, V. (1981) A theory of synaptic interactions. In W. Reichardt and T. Poggio (Eds.), *Theoretical Approaches in Neurobiology*, MIT Press, Cambridge, MA pp. 28-38.
- Rall, W. (1959) Branching dendritic trees and motoneuron membrane resistivity, *Exp. Neurol.*, 1: 491-527.
- Rall, W. (1964) Theoretical significance of dendritic trees for neuronal input-output relations. In R.F. Reiss (Ed.), *Neural Theory and Modeling*, Stanford University Press, Stanford.
- Rall, W. (1977) Core conductor theory and cable properties of neurons. In E. Kandel and S. Geiger (Eds.), *Handbook of Physiology*, American Physiological Society, Washington, D.C.
- Sabah, N.H. and Leibovic, K.N. (1969) Subthreshold oscillatory responses of the Hodgkin-Huxley cable model for the squid giant axon, *Biophys. J.*, 9: 1206-1222.
- Segev, I., Fleshman, J.W., Miller, J.P. and Bunow, B. (1984) Modeling the electrical behaviour of passive neurons with a network analysis program, in preparation.
- Torre, V. and Owen, W.G. (1983) High-pass filtering of small signals by the rod network in the retina of the toad, *Bufo marinus*, *Biophys. J.*, 41: 305-324.
- Torre, V. and Poggio, T. (1978) A synaptic mechanism possibly underlying directional selectivity to motion, *Proc. roy. Soc. B*, 202, 409-416.