Modeling the max Operation

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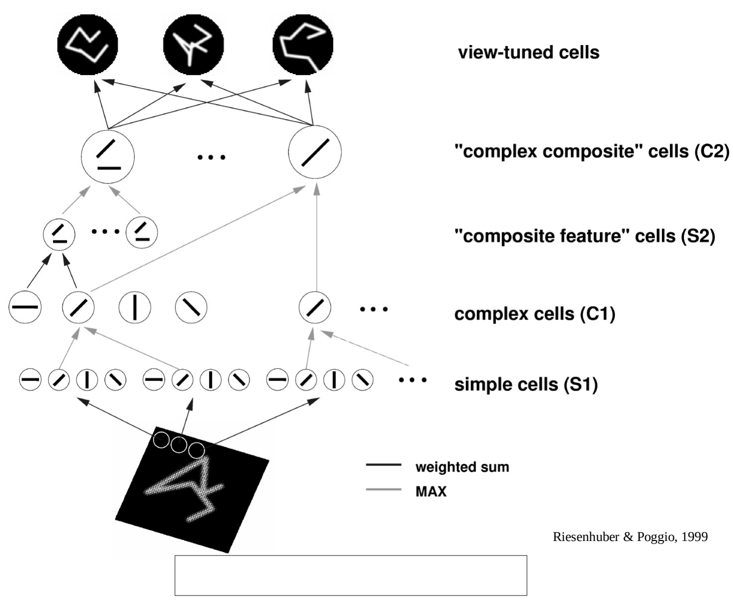
CBCL MIT

Model Diagram



Demystifying the Max Operation

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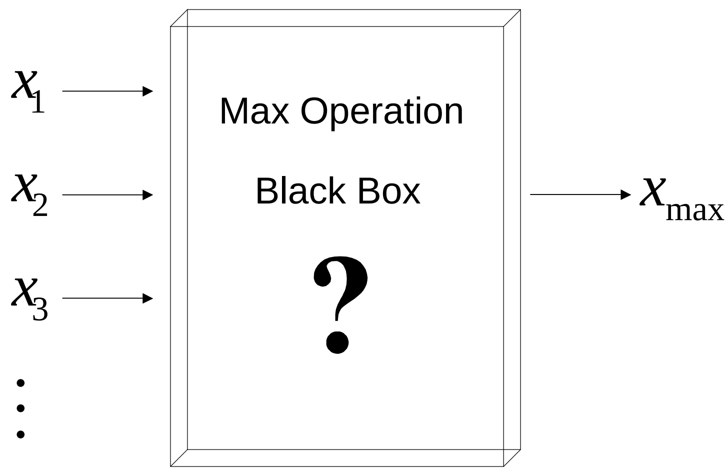

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Schematics: FFN

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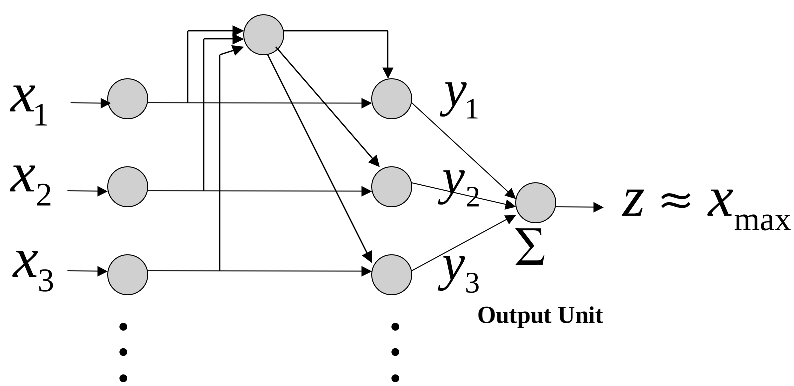
Feedforward Network



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Schematics:FBN

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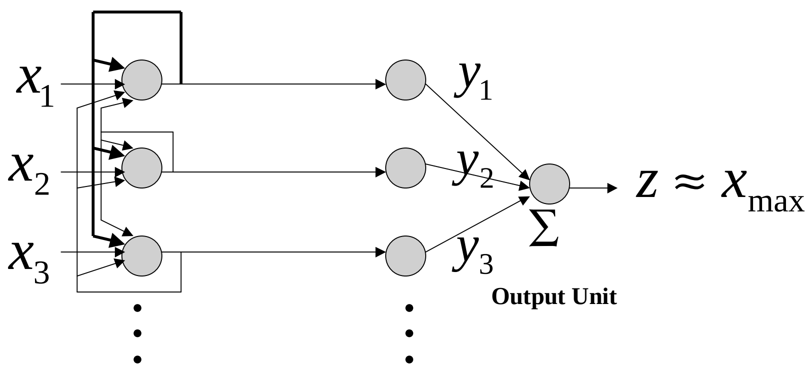
Feedback Network



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Stationary Input

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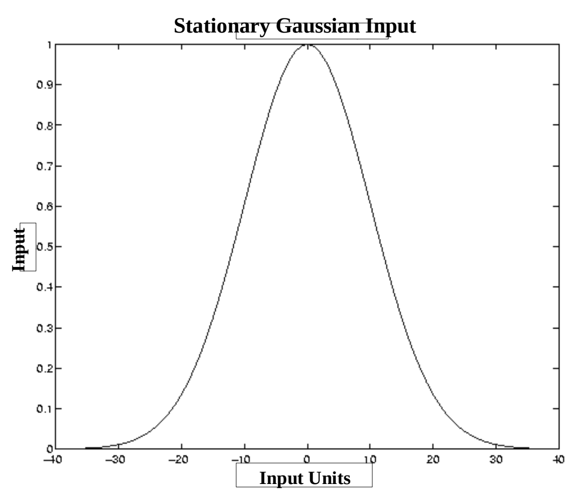

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Feed-Forward Mechanism

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$$y_i = \frac{x_i \cdot f(x_i)}{C + \sum_{j=1}^{n} f(x_j)} \qquad z = \sum_{1}^{n} y_i$$

Given: $X = \{x_1, x_2, ..., x_n\}$ the inputs

C a small constant

f a highly nonlinear, monotonically increasing transfer function

Compute: $Y = \{y_1, y_2, ..., y_n\}$ the hidden layer output

z the final output

Simulating FFN: Exponential *f*

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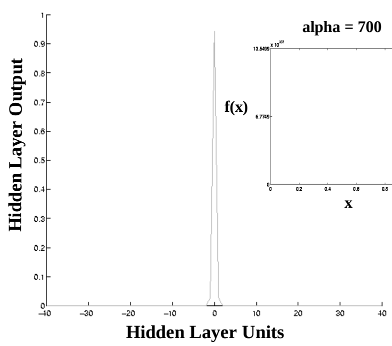
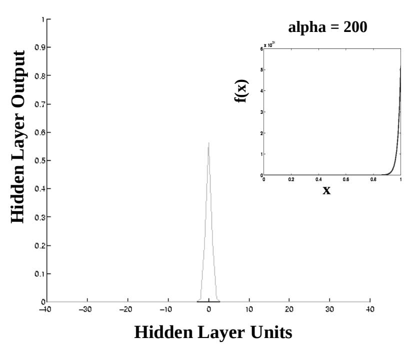
f(x) = exp (alpha * x), alpha = 700



f(x) = exp (alpha * x), alpha =200



Feedback with Divisive Normalization

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Network Dynamics:

$$\tau \cdot y_i'(t) = -y_i(t) + \frac{x_i(t) \cdot f(x_i(t))}{C + \sum_{j=1}^{n} f(y_j(t))}$$

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Simulating FBN with Divisive Normalization

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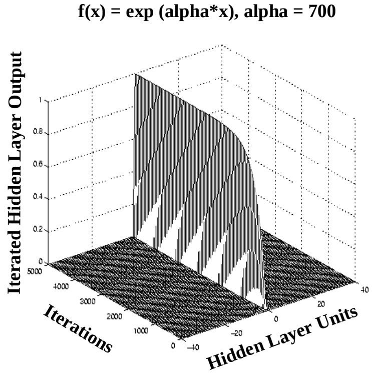
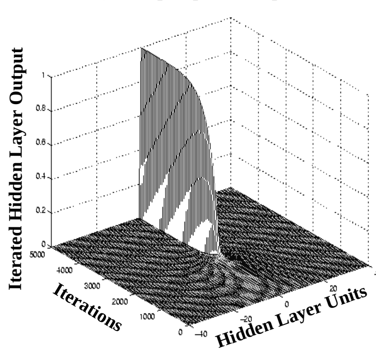
f(x) = exp (alpha*x), alpha = 700



f(x) = exp (alpha*x), alpha = 50



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Linear Feedback Network

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Network Dynamics:

$$\tau \cdot y_i'(t) = -y_i(t) + x_i(t) - k \cdot \sum_{j=1}^{n} [y_j(t)]_+$$

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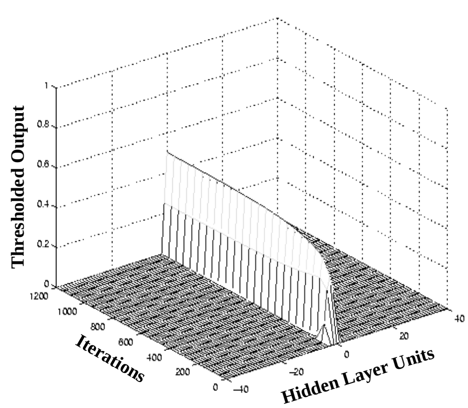
Linear Feedback Network

CBCL MIT

Hidden Layer Output (Post-Threshold)



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Stable Output of Linear FBN

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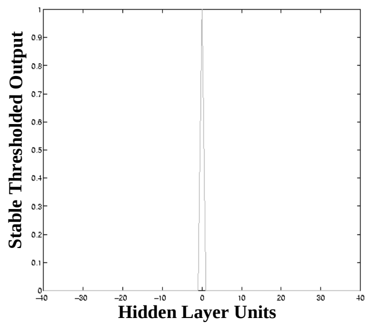
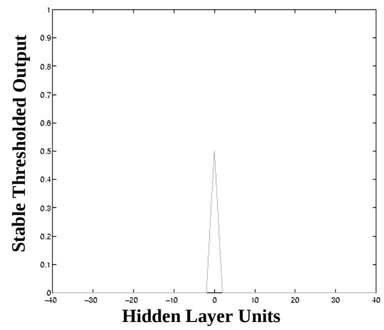
Post-Threshold Output: k = 400 (Spiking Pattern)



Post-Threshold Output: k = 50 (Spiking Pattern)



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Spiking Feedback Network

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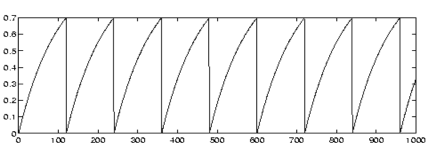
Network Dynamics:

$$\tau \cdot m_i'(t) = -m_i(t) + x_i(t) - k \sum_{j \neq i} y_j(t)$$

$\theta = 0.7$

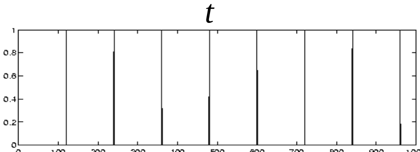
$m_i(t)$



t

$$m_j(t^+) = \begin{cases} 0 & \textbf{if } m_j(t^-) \geq \theta \\ m_j(t^-) & \textbf{otherwise} \end{cases}$$

$y_i(t)$



t

$$y_j(t^+) = \begin{cases} 1 & \textbf{if } m_j(t^-) \geq \theta \\ 0 & \textbf{otherwise} \end{cases}$$

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Spiking Feedback Network

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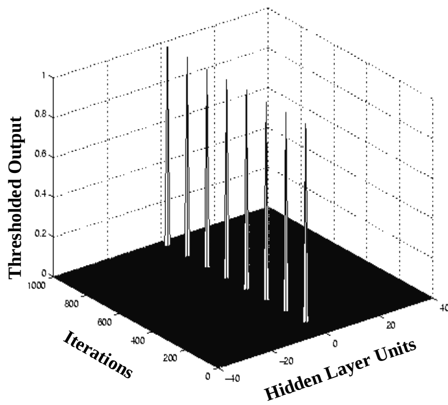
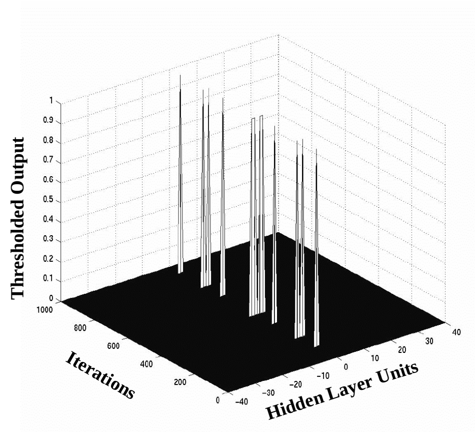
Iterated Spiking Pattern: k = 400



Iterated Spiking Pattern: k = 50



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Summed Output of Spiking FBN

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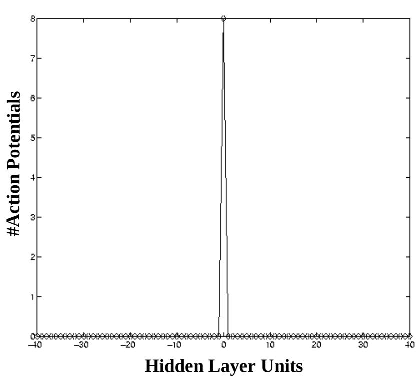
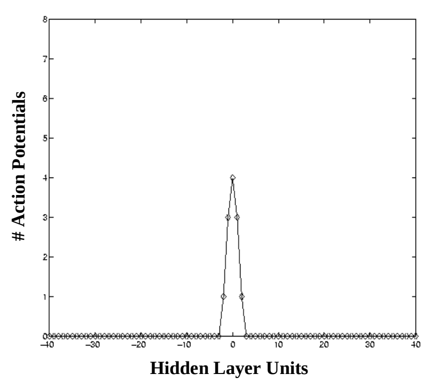
Spike Rate for k = 400



Spike Rate for k = 50



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Robustness w.r.t. Connectivity Strength

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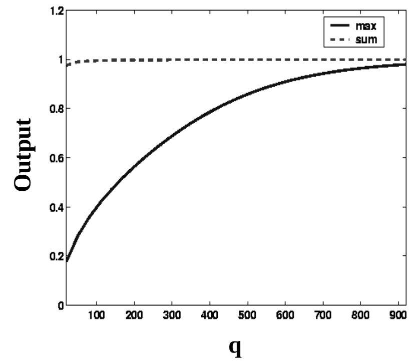
FFN: polynomial



Robustness w.r.t. Input Magnitude

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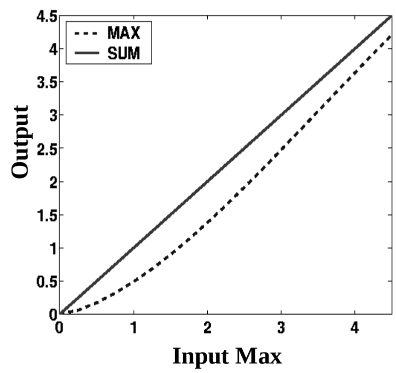
FFN : exponential



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Conclusion

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- ***Max Operation is realizable via various simple networks:***

  >> Feedforward vs. Feedback

  >> Mean Firing Rate vs. Spiking

- ***Simulated networks have the properties:***

  >> Select the max

  >> Preserve input amplitude

  >> Robust w.r.t. connectivity strength

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Outlook

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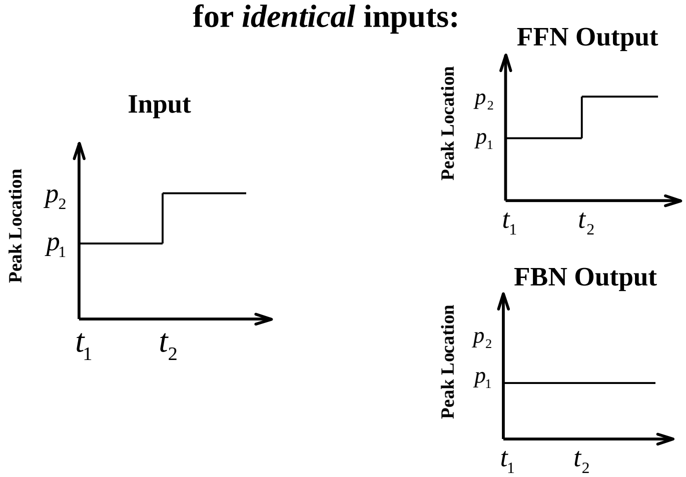
Different models yield *different* predictions for *identical* inputs:



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